

Table 8.3

LEGENDRE FUNCTION—SECOND KIND $Q_n(x)$

x	$Q_0(x)$	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$	$Q_4(x)$	$Q_{10}(x)$
0.00	0.00000 000	-1.00000 000	0.00000 000	0.66666 667	-0.40634 921	0.00000 000
0.01	0.01000 033	-0.99990 000	-0.01999 867	0.66626 669	-0.40452 191	-0.04056 181
0.02	0.02000 267	-0.99959 995	-0.03998 933	0.66506 699	-0.39905 538	-0.08068 584
0.03	0.03000 900	-0.99909 973	-0.05996 399	0.66306 829	-0.38999 553	-0.11993 860
0.04	0.04002 135	-0.99839 915	-0.07991 463	0.66027 179	-0.37741 852	-0.15789 513
0.05	0.05004 173	-0.99749 791	-0.09983 321	0.65667 917	-0.36143 026	-0.19414 321
0.06	0.06007 216	-0.99639 567	-0.11971 169	0.65229 261	-0.34216 562	-0.22828 745
0.07	0.07011 467	-0.99509 197	-0.13954 199	0.64711 475	-0.31978 750	-0.25995 321
0.08	0.08017 133	-0.99358 629	-0.15931 602	0.64114 873	-0.29448 565	-0.28879 038
0.09	0.09024 419	-0.99187 802	-0.17902 563	0.63439 817	-0.26647 538	-0.31447 701
0.10	0.10033 535	-0.98996 647	-0.19866 264	0.62686 720	-0.23599 595	-0.33672 259
0.11	0.11044 692	-0.98785 084	-0.21821 885	0.61856 044	-0.20330 891	-0.35527 122
0.12	0.12058 103	-0.98553 028	-0.23768 596	0.60948 299	-0.16869 616	-0.36990 435
0.13	0.13073 985	-0.98300 382	-0.25705 567	0.59964 048	-0.13245 792	-0.38044 330
0.14	0.14092 558	-0.98027 042	-0.27631 958	0.58903 905	-0.09491 050	-0.38675 142
0.15	0.15114 044	-0.97732 893	-0.29546 923	0.57768 532	-0.05638 395	-0.38873 587
0.16	0.16138 670	-0.97417 813	-0.31449 610	0.56558 646	-0.01721 959	-0.38634 905
0.17	0.17166 666	-0.97081 667	-0.33339 158	0.55275 016	+0.02223 260	-0.37958 962
0.18	0.18198 269	-0.96724 312	-0.35214 699	0.53918 465	0.06161 670	-0.36850 308
0.19	0.19233 717	-0.96345 594	-0.37075 353	0.52489 868	0.10057 361	-0.35318 198
0.20	0.20273 255	-0.95945 349	-0.38920 232	0.50990 155	0.13874 395	-0.33376 565
0.21	0.21317 135	-0.95523 402	-0.40748 439	0.49420 314	0.17577 093	-0.31043 947
0.22	0.22365 611	-0.95079 566	-0.42559 062	0.47781 388	0.21130 336	-0.28343 378
0.23	0.23418 947	-0.94613 642	-0.44351 180	0.46074 476	0.24499 861	-0.25302 221
0.24	0.24477 411	-0.94125 421	-0.46123 857	0.44300 738	0.27652 557	-0.21951 969
0.25	0.25541 281	-0.93614 680	-0.47876 145	0.42461 393	0.30556 765	-0.18327 994
0.26	0.26610 841	-0.93081 181	-0.49607 081	0.40557 719	0.33182 571	-0.14469 251
0.27	0.27686 382	-0.92524 677	-0.51315 685	0.38591 059	0.35502 089	-0.10417 949
0.28	0.28768 207	-0.91944 902	-0.53000 962	0.36562 819	0.37489 746	-0.06219 173
0.29	0.29856 626	-0.91341 578	-0.54661 900	0.34474 467	0.39122 551	-0.01920 468
0.30	0.30951 960	-0.90714 412	-0.56297 466	0.32327 542	0.40380 351	+0.02428 610
0.31	0.32054 541	-0.90063 092	-0.57906 608	0.30123 647	0.41246 080	0.06776 975
0.32	0.33164 711	-0.89387 293	-0.59488 256	0.27864 459	0.41705 981	0.11072 534
0.33	0.34282 825	-0.88686 668	-0.61041 313	0.25551 723	0.41749 822	0.15262 723
0.34	0.35409 253	-0.87960 854	-0.62564 662	0.23187 261	0.41371 084	0.19295 076
0.35	0.36544 375	-0.87209 469	-0.64057 159	0.20772 970	0.40567 128	0.23117 811
0.36	0.37688 590	-0.86432 108	-0.65517 633	0.18310 825	0.39339 336	0.26680 432
0.37	0.38842 310	-0.85628 345	-0.66944 887	0.15802 883	0.37693 227	0.29934 337
0.38	0.40005 965	-0.84797 733	-0.68337 690	0.13251 285	0.35638 546	0.32833 437
0.39	0.41180 003	-0.83939 799	-0.69694 784	0.10658 256	0.33189 317	0.35334 774
0.40	0.42364 893	-0.83054 043	-0.71014 872	0.08026 114	0.30363 867	0.37399 123
0.41	0.43561 122	-0.82139 940	-0.72296 624	0.05357 267	0.27184 811	0.38991 596
0.42	0.44769 202	-0.81196 935	-0.73538 670	+0.02654 221	0.23679 006	0.40082 218
0.43	0.45989 668	-0.80224 443	-0.74739 600	-0.00080 418	0.19877 461	0.40646 477
0.44	0.47223 080	-0.79221 845	-0.75897 958	-0.02843 939	0.15815 208	0.40665 845
0.45	0.48470 028	-0.78188 487	-0.77012 243	-0.05633 524	0.11531 136	0.40128 259
0.46	0.49731 129	-0.77123 681	-0.78080 904	-0.08446 239	0.07067 773	0.39028 551
0.47	0.51007 034	-0.76026 694	-0.79102 336	-0.11299 034	+0.02471 030	0.37368 827
0.48	0.52298 428	-0.74896 755	-0.80074 877	-0.14128 732	-0.02210 100	0.35158 779
0.49	0.53606 034	-0.73733 044	-0.80996 804	-0.16992 027	-0.06923 897	0.32415 933
0.50	0.54930 614	-0.72534 693	-0.81866 327	-0.19865 477	-0.11616 303	0.29165 814

$$Q_0(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

$$Q_1(x) = \frac{x}{2} \ln \left(\frac{1+x}{1-x} \right) - 1$$

$$Q_2(x) = \frac{3x^2-1}{4} \ln \left(\frac{1+x}{1-x} \right) - \frac{3x}{2}$$

$$Q_3(x) = \frac{x}{4} (5x^2-3) \ln \left(\frac{1+x}{1-x} \right) - \frac{5x^2}{2} + \frac{2}{3}$$

$$(n+1)Q_{n+1}(x) = (2n+1)xQ_n(x) - nQ_{n-1}(x)$$

$Q_0(x) = \operatorname{arctanh} x$ (Table 4.17) is included here for completeness.

LEGENDRE FUNCTION—SECOND KIND $Q_n(x)$

Table 8.3

x	$Q_0(x)$	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$	$Q_4(x)$	$Q_5(x)$	$Q_{10}(x)$
0.50	0.54930 614	-0.72534 693	-0.81866 327	-0.19865 477	-0.11616 303	+0.29165 814	
0.51	0.56272 977	-0.71300 782	-0.82681 587	-0.22745 494	-0.16231 372	0.25442 027	
0.52	0.57633 975	-0.70030 333	-0.83440 647	-0.25628 339	-0.20711 759	0.21286 243	
0.53	0.59014 516	-0.68722 307	-0.84141 492	-0.28510 113	-0.24999 263	0.16748 087	
0.54	0.60415 560	-0.67375 597	-0.84782 014	-0.31386 748	-0.29035 406	0.11884 913	
0.55	0.61838 131	-0.65989 028	-0.85360 014	-0.34253 994	-0.32762 069	0.06761 470	
0.56	0.63283 319	-0.64561 342	-0.85873 186	-0.37107 413	-0.36122 172	+0.01449 441	
0.57	0.64752 284	-0.63091 198	-0.86319 116	-0.39942 362	-0.39060 386	-0.03973 144	
0.58	0.66246 271	-0.61577 163	-0.86695 267	-0.42753 983	-0.41523 901	-0.09422 630	
0.59	0.67766 607	-0.60017 702	-0.86998 970	-0.45537 186	-0.43463 218	-0.14810 594	
0.60	0.69314 718	-0.58411 169	-0.87227 411	-0.48286 632	-0.44832 986	-0.20044 847	
0.61	0.70892 136	-0.56755 797	-0.87377 622	-0.50996 718	-0.45592 864	-0.25030 577	
0.62	0.72500 509	-0.55049 685	-0.87446 461	-0.53661 553	-0.45708 410	-0.29671 648	
0.63	0.74141 614	-0.53290 783	-0.87430 597	-0.56274 938	-0.45151 989	-0.33872 031	
0.64	0.75817 374	-0.51476 880	-0.87326 492	-0.58830 338	-0.43903 693	-0.37537 391	
0.65	0.77529 871	-0.49605 584	-0.87130 380	-0.61320 855	-0.41952 271	-0.40576 815	
0.66	0.79281 363	-0.47674 300	-0.86838 239	-0.63739 196	-0.39296 048	-0.42904 673	
0.67	0.81074 313	-0.45680 211	-0.86445 768	-0.66077 634	-0.35943 834	-0.44442 606	
0.68	0.82911 404	-0.43620 245	-0.85948 352	-0.68327 969	-0.31915 810	-0.45121 636	
0.69	0.84795 576	-0.41491 053	-0.85341 027	-0.70481 480	-0.27244 363	-0.44884 377	
0.70	0.86730 053	-0.39288 963	-0.84618 438	-0.72528 868	-0.21974 878	-0.43687 329	
0.71	0.88718 386	-0.37009 946	-0.83774 785	-0.74460 199	-0.16166 443	-0.41503 236	
0.72	0.90764 498	-0.34649 561	-0.82803 775	-0.76264 823	-0.09892 467	-0.38323 471	
0.73	0.92872 736	-0.32202 902	-0.81698 546	-0.77931 296	-0.03241 178	-0.34160 431	
0.74	0.95047 938	-0.29664 526	-0.80451 593	-0.79447 280	+0.03684 038	-0.29049 884	
0.75	0.97295 507	-0.27028 369	-0.79054 669	-0.80799 424	0.10764 474	-0.23053 218	
0.76	0.99621 508	-0.24287 654	-0.77498 679	-0.81973 225	0.17866 149	-0.16259 543	
0.77	1.02032 776	-0.21434 763	-0.75773 539	-0.82952 866	0.24840 151	-0.08787 565	
0.78	1.04537 055	-0.18461 097	-0.73868 011	-0.83721 016	0.31523 275	-0.00787 146	
0.79	1.07143 168	-0.15356 897	-0.71769 507	-0.84258 586	0.37739 063	+0.07559 560	
0.80	1.09861 229	-0.12111 017	-0.69463 835	-0.84544 435	0.43299 312	0.16037 522	
0.81	1.12702 903	-0.08710 649	-0.66934 890	-0.84555 002	0.48006 146	0.24398 961	
0.82	1.15681 746	-0.05140 968	-0.64164 264	-0.84263 849	0.51654 781	0.32364 357	
0.83	1.18813 640	-0.01384 678	-0.61130 745	-0.83641 078	0.54037 123	0.39624 661	
0.84	1.22117 352	+0.02578 575	-0.57809 671	-0.82652 589	0.54946 418	0.45844 913	
0.85	1.25615 281	0.06772 989	-0.54172 080	-0.81259 105	0.54183 191	0.50669 726	
0.86	1.29334 467	0.11227 642	-0.50183 576	-0.79414 886	0.51562 828	0.53731 190	
0.87	1.33307 963	0.15977 928	-0.45802 786	-0.77065 991	0.46925 273	0.54659 757	
0.88	1.37576 766	0.21067 554	-0.40979 212	-0.74147 880	0.40147 508	0.53099 253	
0.89	1.42192 587	0.26551 403	-0.35650 171	-0.70582 022	0.31159 776	0.48727 156	
0.90	1.47221 949	0.32499 754	-0.29736 306	-0.66270 962	0.19967 037	0.41282 291	
0.91	1.52752 443	0.39004 723	-0.23134 775	-0.61090 890	+0.06677 934	0.30602 901	
0.92	1.58902 692	0.46190 476	-0.15708 489	-0.54880 000	-0.08454 828	+0.16680 029	
0.93	1.65839 002	0.54230 272	-0.07268 272	-0.47419 336	-0.24975 925	-0.00265 428	
0.94	1.73804 934	0.63376 638	+0.02458 593	-0.38399 297	-0.42137 701	-0.19666 273	
0.95	1.83178 082	0.74019 178	0.13888 288	-0.27356 330	-0.58752 240	-0.40421 502	
0.96	1.94591 015	0.86807 374	0.27707 112	-0.13540 204	-0.72921 201	-0.60564 435	
0.97	2.09229 572	1.02952 685	0.45181 370	+0.04408 092	-0.81464 729	-0.76587 179	
0.98	2.29755 993	1.25160 873	0.69108 487	0.29436 613	-0.78406 452	-0.81720 735	
0.99	2.64665 241	1.62018 589	1.08264 984	0.70624 831	-0.48875 677	-0.59305 105	

1.00 ∞ ∞ ∞ ∞ ∞

$$Q_0(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad Q_1(x) = \frac{x}{2} \ln \left(\frac{1+x}{1-x} \right) - 1$$

$$Q_2(x) = \frac{3x^2-1}{4} \ln \left(\frac{1+x}{1-x} \right) - \frac{3x}{2} \quad Q_3(x) = \frac{x}{4} (5x^2-3) \ln \left(\frac{1+x}{1-x} \right) - \frac{5x^2}{2} + \frac{2}{3}$$

$$(n+1)Q_{n+1}(x) = (2n+1)xQ_n(x) - nQ_{n-1}(x)$$